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Non-trivial projections of the trivial knot

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1. Introduction.

Homma and the author proved in [HO],[HOT] that any 3-bridge knot diagrams of the trivial knot T always waves, but generally speaking there are many knot diagrams of T which have no waves (defined as 0-waves in this paper) (see [Mo], [Och], [Vi]).

It is here shown that there is an knot diagram of T has no n-waves, where n is an non-negative integer. Moreover, we newly define pseud-waves and admissible diagrams and using these concepts an new algorithm is discussed to decide whether any knots are trivial or not. The author would like to thank professor S. Suzuki and Dr. Nakanishi for valuable conversations.

2. n-waves of knot diagrams

Let K be a knot in R^3 and P(K) be a regular projection of K on a 2-plane R^2 in R^3. Then an arc τ in R^2 is called an n-wave if following conditions hold;

(1) $\partial\tau \cap P(K) = \partial\tau$.

(2) τ intersects P(K) transversely at n interior points which are disjoint from double points of P(K).

(3) one of P_1 and P_2, say P_1 is either an overpath or an underpath of P(K), where P_1 and P_2 are two connected components of $P(K)-\tau$. And

if P_1 is overpath (resp. underpath), then τ must also consist of overpath (resp. underpath) with respect of P(K).

(4) n is less than the number of double points of P(K) in P_1. Moreover, such an arc τ in R^2 is called a pseud-wave if the above conditions (1), (2), (3) holds and n equals the number of the double points of P_1. It will noticed that 0-waves are the same with waves in [HO]. Let τ be a n-wave of P(K). Then it is easily seen that $\tau \cup P_2$ is a knot projection of K′ which is simpler than P(K). That is, P(K′) has crossing points less than them of which P(K) has. As result, the existence of n-waves induce an method to simplify knot projections of knots. And so, if knot projections of the trivial knot have always n-waves, then we get a good algorithm to recognizing whether knots are trivial or not. But it is impossible as follows;

Theorem 1. Given any non-negative integer n, there is a knot projections of the trivial knot, which has no n-waves.

To prove the theorem, at first we give a knot projection of T without no 1-waves, as such an example, we give the Figure 1. We constructed such examples by computer to make knot projections and to compute Jones polynomial [J] of them. Next we deform the knot projections given in Figure 1, and get an another knot projection of T as illustrated in Figure 2.

To verify that the knot projections of T illustrated in Figure 2 has no n-waves, it is sufficient to verify that all overpaths and underpaths have no n-waves. It is easily checked by case by case. As what follows, Theorem 1 is true. It will be noticed that the above method to simplify knots by n-waves except 0-waves may increase bridge indexes of knots.

Next we consider another method to simplify knot projections of knots. Let P(K) be an knot projection of a knot K in R^2 in R^3. Then P(K) is called to be called to be admissible if there is a finite sequence of knot projections $P_0, P_1, \ldots, P_m$ such that P_0 is P(K) and that P_i is obtained by deformation of P_{i-1} along a pseud-wave of P_{i-1} $(i = 1, 2, \ldots, m)$ and that P_m has a n-wave. It is easily seen that both of knot projections given by Figure 1 and 2 are admissible. Furthermore we have checked that many examples of knot projections of the trivial knot by computer using Jones polynomials are admissible. The results of such computations are convincing evidence for the truth of the following conjecture; "All knot projections of the trivial knot are admissible".

3. **Remarks.**

(1) S. Suzuki taught me that he also constructed a knot projection of the trivial knot without n-waves as illustrated in Figure 3.
It will be noticed that Suzuki's original example has two 3-waves and so the above one is slightly modified by the author, but it is also admissible.

(2) The author asked Y. Nakanishi about whether a knot projection of non-trivial knot is obtained from one of the trivial knot by a finite sequence of mutations of it or not. This question is negative by Nakanishi's observation.

(3) The author made many examples of 3-bridge knot projections

without 0-waves by computer and computed Jones polynomials of them to get non-trivial Jones polynomials. And so we conjecture that all 3-bridge knots which have non-trivial Jones polynomials are non-trivial. It will be noticed that by [HO] it can be determined whether any 3-bridge knot is trivial or not.

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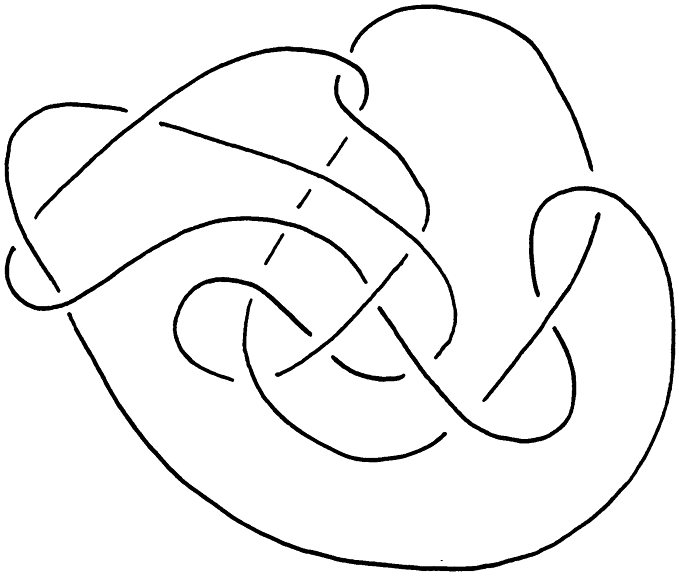

Figure - 1

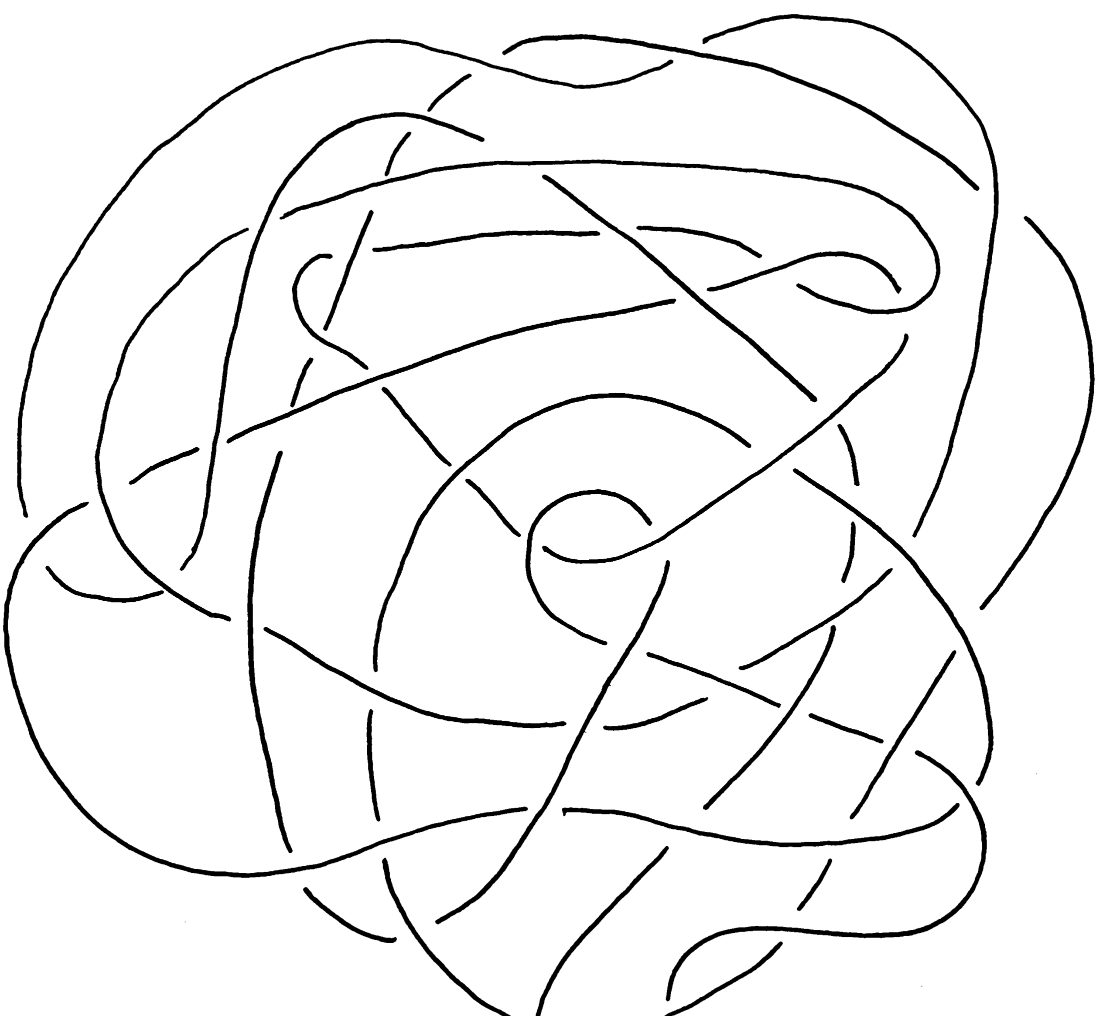

Figure - 2

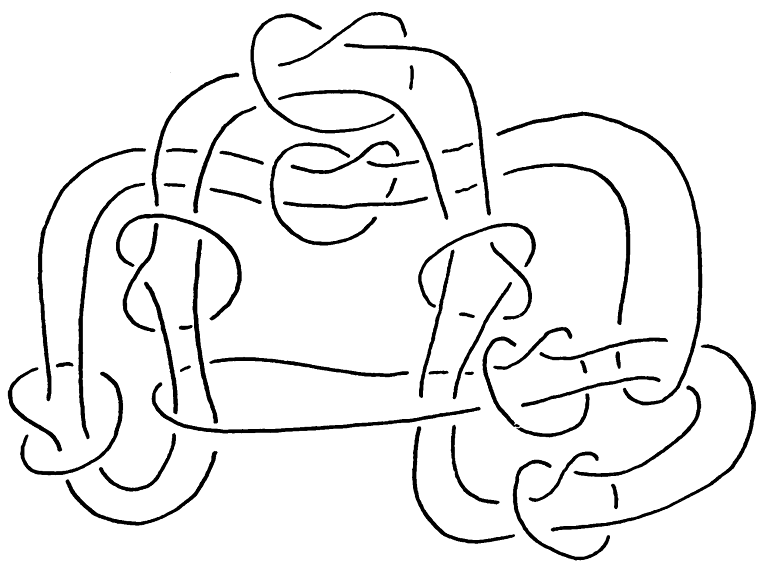

Figure - 3